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Transportation Problem with Pentagonal Intuitionistic Fuzzy Numbers Solved Using Ranking Technique and Russell's Method

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ABSTRACT
This paper presents a solution methodology for transportation problem in an intuitionistic fuzzy environment in which cost are represented by pentagonal intuitionistic fuzzy numbers. Transportation problem is a particular class of linear programming, which is associated with day to day activities in our real life. It helps in solving problems on distribution and transportation of resources from one place to another. The objective is to satisfy the demand at destination from the supply constraints at the minimum transportation cost possible. The problem is solved using a ranking technique called Accuracy function for pentagonal intuitionistic fuzzy numbers and Russell's Method.
An illustrative example is given to verify this approach.

I. INTRODUCTION

The central concept in the problem is to find the least total transportation cost of a commodity. In general, transportation problems are solved with assumptions that the cost, supply and demand are specified in precise manner. However, in many cases the decision maker has no precise information about the coefficient belonging to the transportation problem.An intuitionistic fuzzy set is a powerful tool to deal with such vagueness.

In this paper we introduced pentagonal intuitionistic fuzzy number in a more simplified way which is easy to handle and as a natural interpretation. Under some condition crisp data is insufficient to model the rating of alternatives on attributes in real life decision making problem due to lake of information. In intuitionistic fuzzy set theory the degree of membership (acceptance) and the degree of non-membership (rejection) function are defined simultaneously and their sum of both the values is less than one.

Many authors have shown great interest in intuitionistic fuzzy theory and applied to the field of decision making.In recent years there is more research done to deal with complexity of uncertain data. Here we find the optimal solution for Intuitionist fuzzy transportation problem using the proposed method.

The paper is organized as follows, in section I, introduction with some basic concepts of Intuitionistic Fuzzy numbers have been reviewed, in section II, the proposed algorithm followed by an example and finally section III the conclusion.

I.I. PRELIMINARIES:

1.1.1. Definition: Intuitionistic Fuzzy Set: [5]

Let X be a universal set. An Intuitionistic Fuzzy Set A in X is defined as an object of the form $A^I = \{(x, \mu_{A^I}(x), \vartheta_{A^I}(x)) : x \in X\}$ where the functions $\mu_A : X \to [0,1]$, $\vartheta_A : X \to [0,1]$ define the degree of membership and the degree of non- membership of the element $x \in X$ to the set A^I respectively and for every $x \in X$ in A^I, $0 \le \mu_A(x) + \vartheta_A(x) \le 1$ holds.

1.1.2. Definition: Intuitionistic Fuzzy Number:[5]

An Intuitionistic fuzzy subset $A^I = \{(x_i, \mu_{A^I}(x), \vartheta_{A^I}(x)) / x_i \in X\}$, of the real line R is called an Intuitionistic Fuzzy number if the following holds.

i. There exist $m \in R$, $\mu_{A^I}(m) = 1$ and $\vartheta_{A^I}(m) = 0$, (m is the mean value of A^I).
ii. μ_{A^I} is a continuous mapping from R to the closed interval [0,1] for all $x \in R$, the relation $0 \le \mu_{A^I} + \vartheta_{A^I} \le 1$ holds.

The membership and non-membership function of $\tilde{A}^I$ is of the following form.

$$\mu_{A^I}(x) = \begin{cases} 0 & \text{for } -\propto < x < m - \propto \\ l_1(x) & \text{for } x \in [m - \propto, m] \\ 1 & \text{for } x = m \\ h_1(x) & \text{for } x \in [m, m + \beta] \\ 0 & \text{for } m + \beta \le x < \propto \end{cases}$$

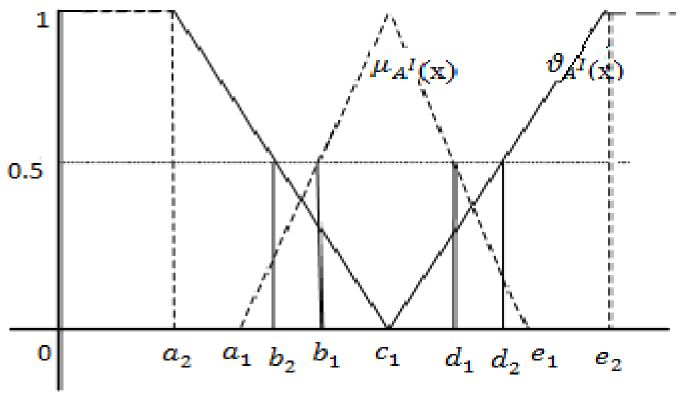
Where l_1(x) and h_1(x) are strictly increasing and decreasing functions in $[m - \propto, ,m]$ and [m, m $+\beta$] respectively.

$$\vartheta_{A^I}(x) = \begin{cases} 1 & \text{for } -\propto < x < m - \propto' \\ l_2(x) & \text{for } x \in [m - \propto', m];\ 0 \le f_1(x) + f_2(x) \le 1 \\ 0 & \text{for } x = m \\ h_2(x) & \text{for } x \in [m, m + \beta'];\ 0 \le h_1(x) + h_2(x) \le 1 \\ 1 & \text{for } m + \beta' \le x \le \propto \end{cases}$$

Here m is the mean value of A^I, $\propto$ *and* β are called left and right spreads of membership and non-membership function of μ_{A^I}(x) respectively. $\propto'$ and β' are called left and right spreads of membership and non-membership function of ϑ_{A^I} (x) respectively.
Symbolically, the Intuitionistic fuzzy number is represented as A^I = (m; $\propto, \beta; \propto', \beta'$)

1.1.3. Definition: Pentagonal Intuitionistic Fuzzy Number (PIFN):[4]

A pentagonal intuitionistic fuzzy number A^I ofan Intuitionistic fuzzy set is defined as A^I = $\{(a_1, b_1, c_1 d_1, e_1)(a_2, b_2, c_2, d_2, e_2)\}$ where all $a_1, b_1, c_1 d_1, e_1, a_2, b_2, c_2, d_2, e_2$ are real numbers and its membership function $\mu_{A^I}(x)$, non-membership function $\vartheta_{A^I}(x)$ are given by



Graphical Representation of Pentagonal Intuitionistic Fuzzy Numbers

$$\mu_{A^I}(\mathrm{x}) = \begin{cases} 0 & \text{for} \quad x < a_1 \\ \frac{x-a_1}{b_1-a_1} & \text{For} \quad a_1 \le x \le b_1 \\ \frac{x-b_1}{c_1-b_1} & \text{For} \quad b_1 \le x \le c_1 \\ 1 & \text{For} \quad x = c_1 \\ \frac{d_1-x}{d_1-c_1} & \text{For} \quad c_1 \le x \le d_1 \\ \frac{e_1-x}{e_1-d_1} & \text{For} \quad d_1 \le x \le e_1 \\ 0 & \text{for} \quad x > e_1 \end{cases}$$

$$\vartheta_{A^I}(\mathrm{x}) = \begin{cases} 1 & \text{for} \quad x < a_1 \\ \frac{b_2-x}{b_2-a_2} & \text{For} \quad a_2 \le x \le b_2 \\ \frac{c_2-x}{c_2-b_2} & \text{For} \quad b_2 \le x \le c_2 \\ 0 & \text{For} \quad x = c_1 \\ \frac{x-c_2}{d_2-c_2} & \text{For} \quad c_2 \le x \le d_2 \\ \frac{x-d_2}{e_2-d_2} & \text{For} \quad d_2 \le x \le e_2 \end{cases}$$

1.1.4Arithmetic Operations of PIFN:[4]

Let A^I={$(a_1, b_1, c_1 d_1, e_1)(\ a_2\ , b_2, c_2, d_2, e_2)$} $and B^I$={$(a_3, b_3, c_3 d_3, e_3)\ (\ a_4\ , b_4, c_4,$ $d_4, e_4)$} be two pentagonal intuitionistic fuzzy numbers, then the arithmetic operations are as follows:

Addition: $\mathrm{A}^{\mathrm{I}}{}_{+}B^I$ = {$(a_1 + a_3, b_1 + b_3, c_1 + c_3, d_1 + d_3, e_1 + e_3)(a_2 + a_4, b_2 + b_4, c_2 + c_4, d_2 + d_4, e_2 + e_4)$}

Subtraction: $\mathrm{A}^{\mathrm{I}} - B^I$ = {$(\ a_1 - e_3, b_1 - d_3, c_1 - c_3, d_1 - b_3, e_1 - a_3)(a_2 - e_4, b_2 - d_4, c_2 - c_4, d_2 - b_4, e_2 - a_4)$}

1.1.5 Ranking of PIFN based on Accuracy Function: [4]

Accuracy function of a pentagonal intuitionistic fuzzy number A^I={$(a_1, b_1, c_1 d_1, e_1)(\ a_2\ , b_2, c_2, d_2, e_2)$} is defined as

$\mathrm{H}(A^I) = (a_1 + a_2 + b_1 + b_2 + c_1 + c_2 + d_1 + d_2 + e_1 + e_2)/5$

1.1.6 Russell's Method:[3]

There are many methods to find the basic feasible solution; one among them is Russell's method for solving transportation problem. Russell's method is probably the better one since it generates a near-optimal initial feasible solution. Here in this paper Russell's method is suitably modified and used for solving Intuitionistic fuzzy transportation problem.

II. PROPOSED ALGORITHM

1. In Intuitionistic Fuzzy Transportation Problem, the quantities are reduced into an integer using the ranking method called Accuracy Function.
2. In the reduced IFTP, identify the row and column difference considering the least two numbers of the respective row and column.
3. Select the maximum among the difference (if more than one, then selects any one) and allocate the respective demand/supply to the minimum value of the corresponding row or column.
4. We take a difference of the corresponding supply and demand of the allocated cell which leads either of one to zero, eliminating corresponding row or column (eliminate both row and column if both demand and supply is zero)
5. Repeat the steps 2, 3 and 4 until all the rows and columns are eliminated.
6. Finally total minimum cost is calculated as sum of the product of the cost and the allocated value.

2.1. AN ILLUSTRATIVE EXAMPLE:

Consider a 3 X 3Pentagonal Intuitionistic Fuzzy Number:

	D1	D2	D3	SUPPLY
S1	[(1,3,5,7,10) (0,3,4,6,11)]	[(2,4,5,9,11) (0,2,6,9,12)]	[(2,4,8,13,15) (1,3,8,12,14)]	25
S2	[(1,3,6,8,9) (1,3,5,9,10)]	[(3,5,7,10,12) (2,4,8,10,14)]	[(2,4,7,9,13) (1,3,6,8,12)]	30
S3	[(2,4,7,10,12) (3,5,7,9,11)]	[(4,7,9,12,15) (3,6,9,11,14)]	[(3,5,6,8,9) (2,4,5,7,11)]	40
DEMAND	35	45	15	

Since $\sum Demand = \sum Supply,$ the problem is a balanced transportation problem.Using the proposed algorithm, the solution of the problem is as follows:
Applying Accuracy function on pentagonal intuitionistic fuzzy number
[(1, 3, 5, 7, 10) (0, 3, 4, 6, 11)], we have
H(A^I) $= (1+0+3+3+5+4+7+6+10+11)/5$= 10
Similarly applying for all the values, we have the following reduced table

2.2 REDUCED TABLE:

	D1	D2	D3	Supply
S1	10	12	16	25
S2	11	15	13	30
S3	14	18	12	40
Demand	35	45	15	

Solution: Step 1

	D1	D2	D3	Supply	Row Difference
S1	10	12 45	16	~~25~~ 0	2
S2	11	15	13	30	2
S2	14	18	12	40	2
Demand	35	~~45~~20	15		
Column Difference	1	3	1	Eliminate Row S1	

Continuing in the same manner, we get the optimal solution as follows:

The Optimal Solution is:

12(45) + 11(35) + 15(20) + 14(25) + 12(15)= 1775

III. CONCLUSION

A method for finding optimal solution in an Intuitionistic fuzzy environmenthas been proposed. We have used Accuracy Function ranking method and Russell's method to find the optimal solution for Pentagonal Intuitionistic Transportation Problem. Thus this method provides an applicable optimal solution which helps the decision maker while they are handling real life transportation problem having Intuitionistic Fuzzy Parameters.

REFERENCES:

[1.] Fuzzy sets and K. Atanassov. 1989. More on Intuitionistic Fuzzy Sets, Fuzzy sets and systems, 33, pp. 37-46
[2.] Atanassov, K.T. "Intuitionistic Fuzzy Sets", Fuzzy sets and systems, Vol.20 (1), pp: 87-96, (1986).
[3.] S. Narayanamoorthy, S.SaranyaandS.Maheswari, "A Method for Solving Fuzzy Transportation Problem (FTP) using Fuzzy Russell's Method ", International Journal of Intelligent Systems and Applications, Vol. 5, No. 2,ISSN: 2074-904,pp: 71-75, (2013).
[4.] Ponnivalavan K. and PathinathanT."Intuitionistic Pentagonal Fuzzy Number", ARPN Journal of Engineering and Applied Sciences, Vol. 10, No. 12, ISSN 1819-6608, pp: 5446-5450, (2015).
[5.] K.Pramila and G.Uthra, "Optimal Solution of an Intuitionistic Fuzzy Transportation Problem", Annals of Pure and Applied Mathematics, Vol. 8, No. 2, ISSN: 2279-087, pp: 67-73, (2014).